

Complex numbers

$$\sqrt{-1} = i$$
$$i^2 = -1$$

Numbers of the form ai , where a is a real number, are called **imaginary numbers**, for example $-5i$ or $\sqrt{3}i$

Numbers of the form $a+bi$, where a and b are both real numbers, are called **complex numbers**, for example $2-3i$ or $-0.5+3\sqrt{2}i$

Complex numbers are often given the symbol z

If $z = a+bi$ then its **conjugate** is given the symbol z^* and is defined by $z^* = a-bi$

z multiplied by z^* is always a real number:

$$(a+bi)(a-bi) = a^2 - (bi)^2 = a^2 + b^2$$

If two complex numbers are equal then the real parts are equal to each other, and also the imaginary parts are equal to each other. We can form two separate equations by **equating real and imaginary parts**

If a quadratic equation has real coefficients and $b^2 - 4ac < 0$ then its two roots are complex numbers, which are always conjugates of each other

Operations with Complex Numbers

Simplify.

1) $i + 6i$

2) $3 + 4 + 6i$

3) $3i + i$

4) $-8i - 7i$

5) $-1 - 8i - 4 - i$

6) $7 + i + 4 + 4$

7) $-3 + 6i - (-5 - 3i) - 8i$

8) $3 + 3i + 8 - 2i - 7$

9) $4i(-2 - 8i)$

10) $5i \times -i$

11) $5i \times i \times -2i$

12) $-4i \times 5i$

13) $(-2 - i)(4 + i)$

14) $(7 - 6i)(-8 + 3i)$

15) $7i \times 3i(-8 - 6i)$

16) $(4 - 5i)(4 + i)$

17) $(2 - 4i)(-6 + 4i)$

18) $(-3 + 2i)(-6 - 8i)$

19) $(8 - 6i)(-4 - 4i)$

20) $(1 - 7i)^2$

21) $6(-7 + 6i)(-4 + 2i)$

22) $(-2 - 2i)(-4 - 3i)(7 + 8i)$

23) $5i + 7i \cdot i$

24) $(6i)^3$

25) $6i \times -4i + 8$

26) $-6(4 - 6i)$

27) $(8 - 3i)^2$

28) $3 + 7i - 3i - 4$

29) $-3i \cdot 6i - 3(-7 + 6i)$

30) $-6i(8 - 6i)(-8 - 8i)$

Critical thinking questions:

31) How are the following problems different?

Simplify: $(2 + x)(3 - 2x)$

Simplify: $(2 + i)(3 - 2i)$

32) How are the following problems different?

Simplify: $2 + x - (3 - 2x)$

Simplify: $2 + i - (3 - 2i)$

Properties of Complex Numbers

Find the absolute value of each complex number.

1) $|7 - i|$

2) $|-5 - 5i|$

3) $|-2 + 4i|$

4) $|3 - 6i|$

5) $|10 - 2i|$

6) $|-4 - 8i|$

7) $|-4 - 3i|$

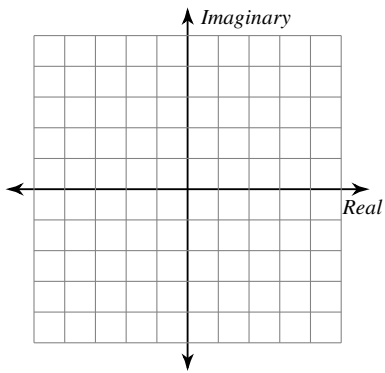
8) $|8 - 3i|$

9) $|1 - 8i|$

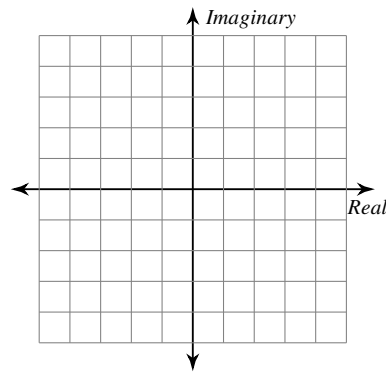
10) $|-4 + 10i|$

Graph each number in the complex plane.

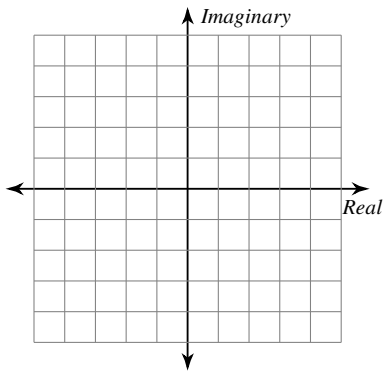
11) $-3 + 4i$



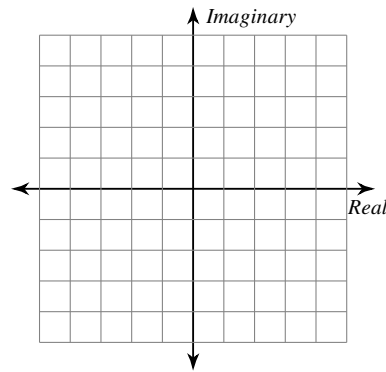
12) $-1 + 5i$



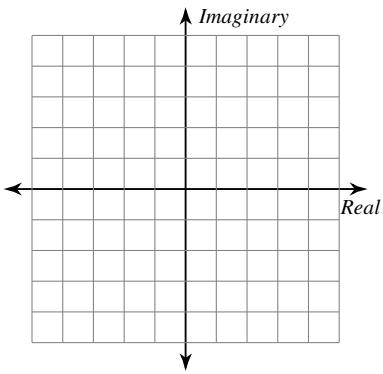
13) $-1 - 4i$



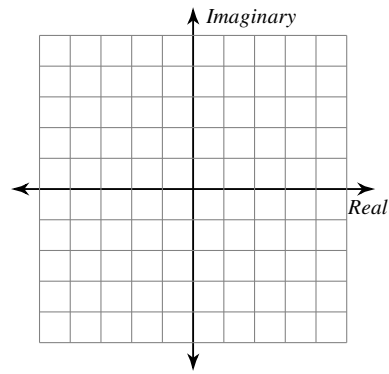
14) $4 + 4i$



15) $-3 + 5i$

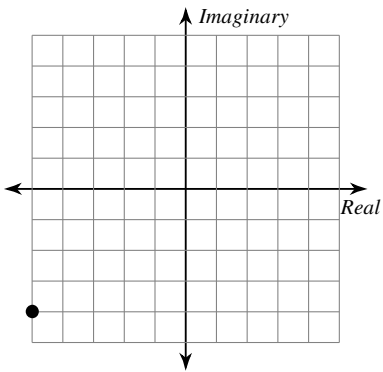


16) $2 + 4i$

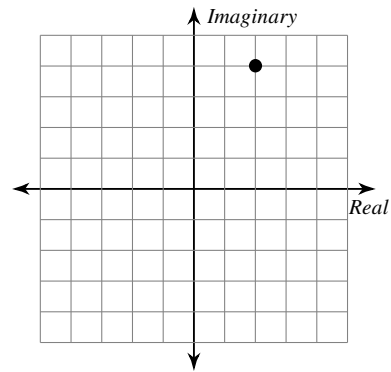


Identify each complex number graphed.

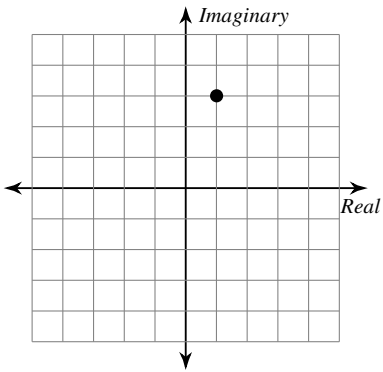
17)



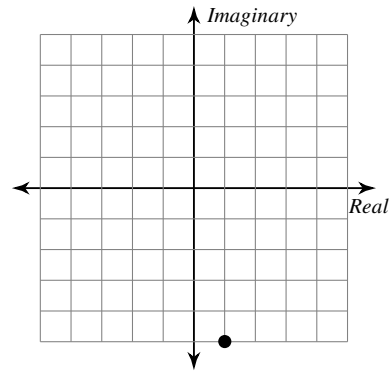
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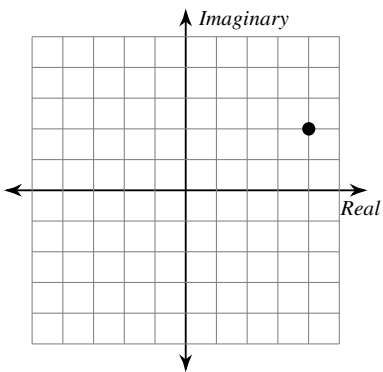
19)



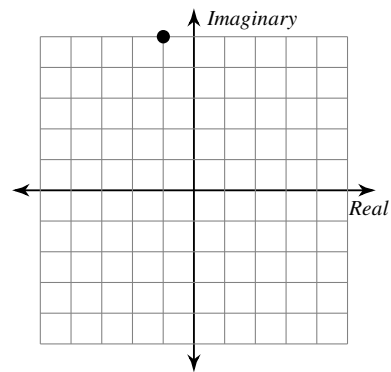
20)



21)



22)



Rationalizing Imaginary Denominators

Simplify.

$$1) \frac{2}{8i}$$

$$2) \frac{3}{5i}$$

$$3) \frac{-5}{-5i}$$

$$4) \frac{-1}{-9i}$$

$$5) \frac{6}{-4i}$$

$$6) \frac{6 + 8i}{9i}$$

$$7) \frac{4 - 9i}{-6i}$$

$$8) \frac{-3 + 10i}{-6i}$$

$$9) \frac{-1 + 8i}{-i}$$

$$10) \frac{10 - 10i}{-5i}$$

$$11) \frac{5i}{-2 - 6i}$$

$$12) \frac{8i}{-1 + 3i}$$

$$13) \frac{1}{-8 - 5i}$$

$$14) \frac{i}{-2 - 8i}$$

$$15) \frac{4}{-3 - 6i}$$

$$16) \frac{-10 - 5i}{-6 + 6i}$$

$$17) \frac{-5 - 9i}{9 + 8i}$$

$$18) \frac{-4 + 10i}{3 + 4i}$$

$$19) \frac{-5 - 3i}{7 - 10i}$$

$$20) \frac{-3 - 7i}{7 + 10i}$$

$$21) \frac{-1 + i}{-5i}$$

$$22) \frac{-6 - i}{i}$$

$$23) \frac{2 + 5i}{-i}$$

$$24) \frac{-4 - 4i}{4i}$$

$$25) \frac{3}{-i}$$

$$26) \frac{a}{ib}$$